Research Article

STABILITY OF NETWORKED BASED THERMAL CONTROL SYSTEM WITH TIME-INVARIANT DELAYS
Venkatachalam V 1,2, Prabhakaran D 1, Thirumarimurugan M 1, Ramakrishnan K 2
1Department of Chemical Engineering, Coimbatore Institute of Technology, Coimbatore, India
2Department of Electrical & Electronics Engineering, Pondicherry Engineering College, Puducherry, India
*Corresponding Author Email: vikrant6488@gmail.com

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ABSTRACT

In this paper, the stability analysis of thermal control system with constant time-delay was used to analyze. In the analysis, PI controller is the essential part of the thermal control system, time-invariant feedback loop delays and then new stability criteria was identified. First, the system was mathematically formulated as a linear retarded continuous-time delayed differential equation with incremental system variables (state variables). Subsequently, by using the stability criterion’s, maximum allowable bound of the network delay that the closed-loop system which can accommodate without losing stability, were computed for various subsets of the controller parameters (PI-controller). The obtained results are the more realistic operating condition in a real time temperature control system. The effectiveness of the proposed result is validated on a benchmark thermal control system.

Keywords: Delay-dependent Stability, Decomposition approach, Linear Matrix Inequality (LMI), Lyapunov-Krasovskii (LK) functional, thermal control, Time-invariant delay, Wirtinger Inequality.

INTRODUCTION

In the scenario of developing high efficient, highly stable and network-based control systems, a systematic and through a study of dynamical system is essential in this way a theoretical study is important to prior design and optimizes the system. The stability analysis of network controlled thermal control system (NCTCS) is very important for all industries, since the heat-exchanger are widely used in space heating, biomedical applications, home appliances (air conditioning, refrigeration), power system generation, chemical, petroleum industries, natural-gas-oil processing, and sewage water refinery.

Thermal system: It is employed to transfer the heat energy between the systems and surrounding. The heat exchanger contains solid wall to prevent mixing of fluids. NCTCS consists of the heat exchanger, sensor, valve, and regulator (PI controller). Due to the network environment, the time delay is inevitable. The heart of the NCTCS has properly tuned PI controller gains with zero network delay. The controller is dedicated to maintaining the output temperature, and it should send the appropriate signal to the actuator. Then only the operator or user can monitor and improve the performance then and there. However, the use of communication link for data transfer in the closed-loop temperature control system introduces delay in the feedback path. The plant or process and controller are connected through a communication link in which the data (control or/measured) experience buffering, processing, and propagation at various points, it may lead to destabilizing the system performance.

In the stability analysis, the state-space technique is a generalized modeling structure for dynamical systems with time-delays. Based on the Lyapunov-Krasovskii functional technique combined with use of the Wirtinger inequality for time-delay systems and a discrete delay decomposition approach to stability, as those methods of stability criterion is presented in LMI framework to compute the maximum value of delay bounds. The proposed stability criterion is expressed as a set of solvable linear matrix inequalities (LMIs) that can be solved using standard numerical packages by causing it as a category of convex optimization problem. Delay-dependent stability criteria can be employed to compute the maximum value of network delays within which the NCTCS remains asymptotically stable. To the best of authors’ knowledge, there is no such research in this field.
SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Figure 1 Block diagram of closed loop temperature control of heat exchanger with time delay

**State space approach:** The mathematical modeling of the heat exchanger (temperature control) system in state space approach, the deviation variables (of the system from their respective equilibrium values) are considered as state variables. The reference input $\theta_{ref}(t)$ is set to zero. The state vector of the closed loop temperature control system is given as:

$$x(t) = \begin{bmatrix} \Delta \theta_1(t) \\ \Delta \theta_2(t) \\ \Delta \theta_3(t) \\ \int \Delta \theta_4(t) dt \end{bmatrix}$$

The corresponding state space model can then be derived easily in following standard framework:

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau),$$

where $x(t) \in \mathbb{R}^{4 \times 1}$ is the state vector of the system; $A \in \mathbb{R}^{4 \times 4}$ and $A_d \in \mathbb{R}^{4 \times 4}$ are system constant matrices. For the temperature control system given in Figure 1, these system matrices are derived as follows:

$$A = \begin{bmatrix} -\frac{1}{T_v} & 0 & 0 & 0 \\ \frac{K_H}{T_H} & -\frac{1}{T_H} & 0 & 0 \\ 0 & \frac{K_F}{T_F} & -\frac{1}{T_F} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The network-induced time delay $\tau = (\tau_1 + \tau_2)$ satisfies the following limiting condition:

$$0 \leq \tau \leq \tau^*$$

where $\tau^* = \max(\tau)$.

Since the temperature control system is modeled as a delayed system, the initial condition is expressed as a continuous-time function as $x(t) = \psi(t), t \in [-\tau^*, 0]$.

**MAIN RESULT**

The proposed stability criterion which has a fewer number of decision factors to the delay-dependent stability of temperature control system (1) and (2) also satisfying the condition (3) by using Lyapunov’s stability theorem. The LMI based stability criterion using the lemmas Jensen integral inequality and Wirtinger inequality are essential and used in the formulation of the following theorems.

**Theorem 1:** For a given non-negative delay $\tau$, the heat exchanger system in (1) and (2) satisfying the condition (3) is asymptotically stable, if there exist real, symmetric, positive definite matrices $P = P^T > 0; S = S^T > 0$ and $R = R^T > 0$; symmetric matrix $Z = Z^T \geq 0$; and free matrix $Q$ such that the following LMIs hold:

$$\begin{align*}
\Pi_1(\tau) &> 0, \\
\Pi_2(\tau) &< 0,
\end{align*}$$

with

$$\begin{align*}
\Pi_1(\tau) &= \begin{bmatrix} P & Q \alpha \tau \\ \ast & Z + \frac{\alpha}{\tau} \end{bmatrix}, \\
\Pi_2(\tau) &= \begin{bmatrix} \Pi_1(\tau) & -\frac{1}{\tau} W(R) \\ \ast & S & \ast & 0 \\ \ast & -S & \ast & 0 \\ \ast & \ast & \ast & 0 \end{bmatrix},
\end{align*}$$

where

$$\begin{align*}
\Pi_{11} &= P A_d - Q - \frac{\alpha}{\tau^2} A_d^T Q - \frac{\alpha}{\tau} A_d^T R, \\
\Pi_{12} &= \begin{bmatrix} \Pi_{11} & \tau A_d^T Q + \tau Z \\ -S & \tau A_d^T Q - \tau Z & 0 \\ 0 & 0 & \tau A_d^T R & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},
\end{align*}$$

The network-induced time delay $\tau = (\tau_1 + \tau_2)$ satisfies the following limiting condition:
Now, if LMI (4) and (5) hold simultaneously, then \( \Pi_3(\tau) < 0 \), this implies that the delayed temperature control system is asymptotically stable that is in accordance with Lyapunov stability theorem\(^{13}\).

**Remark 1:** The proposed theorem 1 is under the category of constrained optimized problem as follows the objective function is\(^{26,27}\):

\[
\begin{bmatrix}
\tau \\
\end{bmatrix}
\in P_{0, Q, R, S},
\text{Subject to}
\Pi_4(\tau) > 0,
\Pi_5(\tau) < 0;
\text{P} = P^T > 0; S = S^T > 0; R = R^T > 0; Q = Q^T.
\]

**Theorem 2:** For a given non-negative delay \( \tau \), and a positive integer \( N \geq 2 \), the heat exchanger system in (1) and (2) satisfying the condition (3) is asymptotically stable\(^{11,12} \), if there exist positive, real, symmetric matrices \( P = P^T > 0; Q = Q^T > 0; R = R^T > 0; W = W^T \geq 0; S_i = S_i^T (i = 1, 2, ..., N) \), such that

\[
S = S^T = \begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1N} \\
S_{21} & S_{22} & \cdots & S_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
S_{N1} & S_{N2} & \cdots & S_{NN}
\end{bmatrix} \geq 0
\]

\[
\Xi = \begin{bmatrix}
\Xi_1 & \Xi_2 & \Xi_3 \\
* & -W & 0 \\
* & * & -R
\end{bmatrix} < 0,
\]

where

\[
\Xi_1, \Xi_2, \Xi_3 \text{ is specified on top of the page}.
\]

\[
\begin{bmatrix}
P \\ Q \\ \tau
\end{bmatrix} \geq 0
\]

\[
\theta(\tau) < 0
\]

**Remark 2:** The proposed theorem 2 is under the category of constrained optimized problem as follows the objective function is\(^{26,27}\):

\[
\begin{bmatrix}
\Xi_1^{(1)} & \Xi_2^{(1)} & S_{12} & \cdots & S_{1N} \\
* & \Xi_2^{(1)} & \Xi_3^{(1)} & \cdots & S_{2N} \\
* & * & \Xi_3^{(1)} & \cdots & S_{N1} \\
* & * & * & \cdots & \Xi_{NN} \\
* & \Xi_1^{(1)} & * & * & \cdots
\end{bmatrix} \Xi^{(2)} = \begin{bmatrix}
\frac{h}{N} A^T \Xi^{(2)} \\
0 \\
\vdots \\
0 \\
\frac{h}{N} B^T \Xi^{(2)}
\end{bmatrix}
\]

with

\[
\Xi_1^{(1)} = A^T P + PA + Q - W - R + S_{11},
\]

\[
\Xi_2^{(1)} = S_{22} - S_{11} - W,
\]

\[
\Xi_3^{(1)} = S_{33} - S_{32},
\]

\[
\Xi_1^{(2)} = W + S_{12}, \Xi_2^{(2)} = S_{23} - S_{22}
\]

**Theorem 3:** For a given non-negative delay \( \tau \), the heat exchanger system in (1) and (2) satisfying the condition (3) is asymptotically stable\(^{11} \), if there exist positive, real, symmetric definite matrices \( P = P^T > 0; S = S^T > 0 \) and \( R = R^T > 0 \); and free matrix \( Z \) such that the following LMIs hold\(^{19,26,27}\):

\[
\begin{bmatrix}
\theta_{11} & PA_d - Q - \tau A_d Q + \tau Z \\
* & -S \tau A_d Q - \tau Z
\end{bmatrix} \geq \begin{bmatrix}
\tau A_d^T R A_d \\
* + \tau A_d^T R A_d
\end{bmatrix} + \begin{bmatrix}
4R \\ 2R \\ -6R \\
* + 4R \\ -6R \\
* + * + 12R
\end{bmatrix}
\]

\[
\theta(\tau) = \begin{bmatrix}
\theta_{11} & PA_d - Q - \tau A_d Q + \tau Z \\
* & -S \tau A_d Q - \tau Z
\end{bmatrix} \geq \begin{bmatrix}
\tau A_d^T R A_d \\
* + \tau A_d^T R A_d
\end{bmatrix} + \begin{bmatrix}
4R \\ 2R \\ -6R \\
* + 4R \\ -6R \\
* + * + 12R
\end{bmatrix}
\]

\[
\theta(\tau) = \begin{bmatrix}
\theta_{11} & PA_d - Q - \tau A_d Q + \tau Z \\
* & -S \tau A_d Q - \tau Z
\end{bmatrix} \geq \begin{bmatrix}
\tau A_d^T R A_d \\
* + \tau A_d^T R A_d
\end{bmatrix} + \begin{bmatrix}
4R \\ 2R \\ -6R \\
* + 4R \\ -6R \\
* + * + 12R
\end{bmatrix}
\]
with \( \theta_{11} = A^T P + PA + S + Q + Q^T \)

**Remark 3:** For the proposed theorem 3, is under the category of constrained optimization problem as follows the objective function \( g^{[26,27]} \),

\[
\max \tau \quad (14)
\]

Subject to

\[
\begin{bmatrix} P & Q \\ I & Z \end{bmatrix} \geq 0, \\
\theta(\tau) < 0 \}
\]

\( P = P^T > 0; S = S^T > 0; R = R^T > 0; Z = Z \geq 0 \)

The convex optimization problem can be solved readily using standard numerical packages \( ^{11,12,26,32} \).

**CASE STUDY AND DISCUSSION**

The presented robust stability criterion’s is validated on temperature control system. The system parameters of the benchmark temperature control system taken from \(^4\) are presented in Table 1.

<table>
<thead>
<tr>
<th>System</th>
<th>Gain</th>
<th>Time-Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat Exchanger</td>
<td>( K_p = 34 )</td>
<td>( T_m = 30 )</td>
</tr>
<tr>
<td>Valve</td>
<td>( K_p = 1 )</td>
<td>( T_e = 0.4 )</td>
</tr>
<tr>
<td>Sensor</td>
<td>( K_p = 0.08 )</td>
<td>( T_e = 2 )</td>
</tr>
</tbody>
</table>

Assuming zero network delay condition, the PI controller capability curve, shown in Figure 2 gives the region for the controller parameters \( K_p \) and \( K_i \) for the asymptotically stability. If \( K_p \) and \( K_i \) are chosen to lie on the capability curve, then the system will have one pair of closed-loop poles on \( j\omega \) axis signifying a marginally stable operating condition. The closed-loop poles of the system for some values of \( K_p \) and \( K_i \) lying on the curve is presented in Table 2. In order to see the effect of time-delay on the performance of the closed-loop control system, a value of \( K_p = 4 \) and \( K_i = 0.2 \) that lies well within the controller capability curve is selected. For these controller parameters, when the closed loop system is simulated for a unit perturbation in heat exchanger output variable \( \Delta h_0(t) \) from its equilibrium value \( (t.e., \Delta h_{eq,0}(t) = 0) \) at the time \( t = 0 \), the incremental variable converges asymptotically to equilibrium point \( \Delta h_0(t) \rightarrow 0 \) as \( t \rightarrow \infty \) as illustrated in Figure 3.

For controller parameter \( K_p = 4 \) and \( K_i = 0.2 \), when the control is implemented in networked environment (with the presence of network delay), according to the presented delay-dependent stability criterion in Theorem 1 (solved by the stability criterion as a convex optimization problem as specified in Remark 1), the temperature control system is asymptotically stable up to the delay \( \tau = 0.8765 \) seconds as shown in Table 3. For the same controller parameter gain and same network condition, according to the delay-dependent stability criterion is presented in theorem 2 (solved by the criterion as under a convex optimization problem as specified in Remark 2), the temperature control system is asymptotically stable up to \( \tau = 0.8766 \) seconds for positive integer \( N = 5 \) as shown in Table 4. In the same theorem for positive integer \( N = 10 \), the temperature control system is asymptotically stable up to \( \tau = 0.8767 \) seconds as shown in Table 5. in the stability theorem 3 is presented for same controller gain parameter, when the controller is implemented in the networked environment (solved by the stability criterion as under a convex optimization problem as specified in Remark 3), the temperature control system is asymptotically stable up to \( \tau = 0.8767 \) seconds as shown in Table 6. The Theorem 3 is getting tighter bounds to compare the Theorem 1&2 are listed in Table 7.

**SIMULATION RESULTS**

In this section, simulation results are presented to substantiate the analytical results. This means that when the network delay is of this value, i.e., \( \tau \leq 0.8767 \text{ secs} \), the closed loop system is stable; if the delay exceeds this critical value, according to the proposed result, the closed loop system becomes unstable. When \( \tau = 0.8767 \text{secs} \), it is observed through the simulation study that the closed loop system is marginally stable i.e. the deviation variable \( \Delta h_0(t) \) exhibits undamped oscillations about the equilibrium point shown in Figure 4.

If the maximum value of the time-delay is increased from \( \tau = 0.8766 \text{ secs} \), then the system becomes unstable; see, Figure 5 for unbounded evolution of state variable \( \Delta h_0(t) \) and when operated at delay less than \( \tau = 0.8767 \text{secs} \), the deviation variable \( \Delta h_0(t) \) converges asymptotically towards the equilibrium state as shown in Figure 6 signifying a stable operating condition. Hence, the simulation results clearly substantifies the theoretical performance by bringing out the effect of time-delay in the closed loop network condition on delay-dependent stability and performance of the benchmark temperature control system.

For the other subsets of \((K_p, K_i)\), the maximum allowable value of network delay provided by the Theorems 1, 2, 3 is given in Table 6. (has the maximum upper bound delay) is stated herewith. Consider the case when \( K_p \) is chosen as 4.0 and \( K_i \) as 0.2. Then according to the stability criterion of LMI based methods, the closed loop temperature control system can withstand a total network-induced delay of \( \tau = 0.8767 \text{secs} \) without losing stability (it is clear from the simulation that as the network delay approaches the critical value of \( \tau = 0.8768 \text{secs} \), the performance of the system is seriously affected). If \( K_i \) is retained at 0.2 and if \( K_p \) is increased to 5.0 (increasing \( K_p \) improves the transient response of the system), the closed loop system withstands only a delay of \( \tau = 0.5285 \text{secs} \). This clearly shows that there is a drastic reduction in the allowable delay value if one attempts to improve the transient response. On the other hand, if \( K_p \) is fixed at 4.0 and if \( K_i \) is increased to 0.3 (increasing \( K_i \) improves the steady-state response of the system), again the maximum allowable bound for the network delay
drops to $\tau = 0.5690 \text{ secs}$. It is clear that an attempt to improve the steady state response is accompanied by a reduction in the allowable margin for the network delay. In this back drop, table 6. Presents maximum stable delay margin for multiple choices of the controller parameters that can be selected depending upon whether the focus is on improved transient response or the improved steady state response.

For the heat exchanger system is considered in this paper, if the network permits a maximum delay of 0.9 secs for signal transfer in the feedback loop, then a reasonable choice of the controller parameters will be $K_I = 0.2$ and $K_P = 4$. For these recommended values, the closed loop system is capable of accommodating a total network delay of $\tau = 0.8767 \text{ secs}$ which is optimal for the given network conditions.

<table>
<thead>
<tr>
<th>$K_P$</th>
<th>$K_I$</th>
<th>0.02</th>
<th>0.04</th>
<th>0.08</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
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<td>*</td>
</tr>
<tr>
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<td>4.3301</td>
<td>0.8533</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td>1.5</td>
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<td>6.1350</td>
<td>4.7180</td>
<td>4.0676</td>
<td>1.6378</td>
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</tr>
<tr>
<td>2.0</td>
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</tr>
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<td>*</td>
</tr>
<tr>
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<td>0.8765</td>
<td>0.5690</td>
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<tr>
<td>4.5</td>
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<td>0.4497</td>
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<tr>
<td>5.0</td>
<td>0.8654</td>
<td>0.8287</td>
<td>0.7547</td>
<td>0.7174</td>
<td>0.5284</td>
<td>0.3386</td>
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</table>
CONCLUSION

In this paper, Lyapunov stability techniques are presented to determine the delay-dependent stability of temperature control system under the network environment. The first method, using Lyapunov-Krasovskii method combined with Wirtinger inequality (Wirtinger inequality for time-delay systems and Application to time-delay systems) and the second method, direct decomposition approach to stability of linear systems, a less conservative stability criterion is presented in LMI framework to compute the stable delay margin. Those methods are validated with a standard benchmark temperature control system and the proposed results are more realistic working condition in a real-time temperature control system.

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REFERENCES


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Table 4: Maximum upper bound delay for different values of $K_p$, $K_I$ and $N = 5$ using decomposition approach

<table>
<thead>
<tr>
<th>$K_p$</th>
<th>0.02</th>
<th>0.04</th>
<th>0.08</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
</tr>
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<td>20.5003</td>
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<td>4.1157</td>
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<td>11.5511</td>
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Table 5: Maximum upper bound delay for different values of $K_p$, $K_I$ and $N = 10$ using decomposition approach

<table>
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<tr>
<th>$K_p$</th>
<th>0.02</th>
<th>0.04</th>
<th>0.08</th>
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<th>0.2</th>
<th>0.3</th>
</tr>
</thead>
<tbody>
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<td>*</td>
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<td>5.6006</td>
<td>4.3349</td>
<td>0.8534</td>
<td>*</td>
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<tr>
<td>1.5</td>
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Table 6: Maximum delay bounds for different values of $K_p$ and $K_I$ using Wirtinger Inequality-2

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